XXIV. *A Diſcourſe concerning the Menſtrual Parallax, ariſing from the mutual Gravitation of the Earth and Moon; it's Influence on the Obſervations of the Sun and Planets; with a Method of obſerving it:* By J. Smeaton, *F. R. S.*

Read May 12, 1768.

IT is demonſtrated by Sir Iſaac Newton in the *Principia*, that it is not the Earth's center, but the *common center of gravity* of the Earth and Moon, that deſcribes the ecliptic; and that the Earth and Moon revolve in ſimilar ellipſes, about their common center of gravity. The ſame great author has alſo inveſtigated, from the different riſe of the tides, when the Moon is in conjunction or oppoſition to the Sun, to thoſe which happen when the Moon is in her quadratures; that the quantity of matter in the Earth is to that in the Moon, as 39. 78 to 1; from whence, and the known diſtance of the Earth and Moon, it would follow, that the common center of gravity of the two bodies falls without the ſurface of the Earth, by one half of its ſemidiameter: that is, that the center of the Earth deſcribes an epicycle round the common center of gravity once a month, whoſe diameter is three ſemidiameters of the Earth.

Dr. Gregory, in his aſtronomy, has laid hold of this circumſtance, in order to prove the relative gravity of the Earth and Moon, by obſervation; which is the ſubject of his 60th propoſition of the fourth book;

book; in which he has demonſtrated, that if an obſerver on the Earth makes a correct obſervation on the Sun's place, when the Moon is in one quadrature, it will differ from a like obſervation, taken in the oppoſite quadrature (according to a mean elliptic motion) by an angle which the diameter of this epicycle will ſubtend at the Sun. The ſame learned author has alſo ſhewn, in the ſcholium to the ſame propoſition, that this quantity, or parallax, will be twice greater to Mars in oppoſition, and three times greater to Venus, in her inferior conjunction with the Sun.

The difference thus produced in the apparent place of the Sun, and of all the primary planets being governed by the Moon, and having it's period the ſame, may perhaps be not unaptly called the *menſtrual parallax*.

Now if, with Sir Iſaac Newton, the relative gravities of the Earth and Moon are taken between the proportion of 39 and 40 to one; the menſtrual parallax of the ſun will come out 13″ upon the radius of the Earth's epicycle, and will affect the ſolar obſervations at the oppoſite quadratures, by double that quantity, viz. 26″: in like manner, the mean diſtance from the Earth of Mars in oppoſition, being to the Sun's mean diſtance, as 1 to 2. 1; and the leaſt diſtance of Mars from the Earth, to the Sun's mean diſtance, as 1 to 2 $\frac{1}{4}$, the menſtrual parallax of Mars will affect the obſervation upon him in that ſituation, by 56″ and 73″ $\frac{1}{2}$, reſpectively.

The mean diſtance of Venus from the Earth, in her inferior conjunction, being to that of the Sun as 3 $\frac{1}{2}$ to 1 nearly, and not very variable, on account of

of the orbit of Venus being almoſt circular; the menſtrual parallax would affect the place of Venus, in that ſituation, by a quantity not leſs than 92″; and in all other ſituations in proportion to her diſtance; which alſo holds with reſpect to all the reſt of the planets.

Theſe diſturbing quantities are by no means to be diſpenſed with, in the nice and critical ſtate that aſtronomical obſervations and calculations have arrived at, in conſequence of the diſcoveries of Dr. Bradley, who may be ſaid to have given a baſis to aſtronomy; however, could we rely upon the *data*, on which Sir Iſaac's inveſtigation of the relative gravity of the Earth and Moon is founded, we ſhould have nothing to do but to apply an equation to the particular caſes, according to the diameter of the epicycle, as deduced from the relative gravity; but whoever conſiders the great obſtructions that the water of the ſea meets with in its motion to obey the influence of the Moon; the great difficulty in aſcertaining the true height of the tides, from the many diſturbing cauſes intervening; and the many uncertainties, and want of coincidence, that have attended, and muſt attend, ſuch obſervations; muſt confeſs, that this matter does not ſeem capable of ſuch a determination from that quarter, as the preſent ſtate of aſtronomy requires.

Accordingly, ſince the time of Dr. Gregory, thoſe great aſtronomers Dr. Bradley, De la Caille, and others, have applied themſelves to determine the quantity of the menſtrual parallax from ſolar obſervations: but though theſe have given cauſe to ſuppoſe that the relative gravity of the Earth and Moon are not above $\frac{2}{3}$ of the quantity deduced from the tides; yet, as the obſervation

obſervation of theſe ſmall angles principally depends upon the obſervation of the Sun's right aſcenſion (which, depending on the meaſure of time, is leſs capable of exact obſervation, than if depending on divided inſtruments); the deductions thence drawn ſeem ſtill wanting of that certainty which the ſubject demands; and if to this we add, from a deduction of Mr. Maſkelyne, that the relative gravity of the Earth and Moon is as 76 to 1, derived from the effect that the Moon produces in the nutation of the Earth's axis; the relative gravity, and conſequently the parallaxes thereon depending, will be reduced to almoſt one half of thoſe reſulting from Sir Iſaac's determination.

It is true, that the quantity of effect of the menſtrual parallaxes will not be great, if computed upon Mr. Maſkelyne's induction, for as much as that the common center of gravity will be conſiderably within the Earth's ſurface; yet, even in that caſe, the Sun's tranſit over the meridian, when the Moon is in one quadrature, will differ nearly one ſecond of time from that obſerved in the oppoſite quadrature; and though De la Caille and Mayer have formed equations depending on the Moon, to be applied to the equation of time; yet, if we are at an uncertainty, whether the *maximum* of this equation is one ſecond, two thirds of a ſecond, or half a ſecond of time, each way, we are ſtill under a material difficulty; for though theſe differences are ſo ſmall, that it is not eaſy to determine them exactly from ſolar obſervations; yet, as they are capable of creating a ſenſible difference in theſe obſervations, they will, ſo long as they remain undetermined, prevent that ſolidity and firmneſs

neſs to the ſolar obſervations, which is the more neceſſary as they are the foundation of all the reſt: but with reſpect to thoſe planets, that in their periods come nearer to us than we to the Sun, the obſervations upon them will be affected by a greater uncertainty.

The determination of the menſtrual parallax is of ſtill more importance, as it is a neceſſary conſideration in the determination of the Sun's parallax; and this, whether deduced from Mars or Venus, as I ſhall preſently ſhew more particularly; but firſt I muſt ſtate the quantity of the menſtrual parallax, according to the beſt *data* yet known, by a contrary proceſs; and, taking the mean quantity of the Sun's parallax, according to the determination of Mr. Short, at 8″ 8, and the relative gravities of the Earth and Moon, according to Mr. Maſkelyne, as 76 to 1, and the mean diſtance of their centers equal to 60 $\frac{1}{2}$ ſemidiameters; we ſhall then have the diſtance of the Earth's center from the center of gravity, at $\frac{8}{10}$ of the Earth's ſemidiameter (that is, $\frac{1}{5}$ of that ſemidiameter within the Earth's ſurface) and the menſtrual parallax equal to $\frac{8}{10}$ of the Sun's parallax; conſequently about 7″; and the double menſtrual parallax, or vacillation, ariſing from the whole diameter of the epicycle, 14″; the mean menſtrual parallax of Mars in oppoſition, 29″ $\frac{1}{2}$; the greateſt, 38″ $\frac{1}{2}$; and that of Venus 49″; from hence it follows, that, was a perſon to attempt the Sun's parallax, by the diurnal motion of the Earth, applied as a baſis to Mars in oppoſition, as has formerly been tried; and ſhould the Moon be at new or full at the ſame time, the change of place of the Earth's center, in its own epicycle, would amount to an angle ſeen from Mars

of

of 1″.3 nearly; that is, in caſe the interval between the obſervations was eight hours, and Mars at his mean diſtance; but if Mars was not at his neareſt diſtance, this change would in the ſame time amount to 1″.7 nearly. In like manner, if a tranſit of Venus happens near the new or full Moon (as will be the caſe next year), the time of the tranſit will be affected by a change of place, ſuch as the Earth's center will deſcribe in its epicycle, during the time of the whole tranſit, if the beginning and end are obſerved in the ſame place; or during the difference of abſolute time, at which the tranſit appears to begin or end to different obſervers in diſtant meridians. Thus, when the ſame obſerver ſees the beginning and end in the ſame place, the baſe deſcribed by that obſerver, from the Earth's diurnal motion, muſt be corrected by the ſpace deſcribed by the Earth's center, in the circumference of it's epicycle, during that time; which, if it be ſuppoſed of ſeven hours, will amount to an angle of 1″.9, ſeen from Venus: but, where the beginning or end is ſeen by different obſervers in diſtant meridians, as the difference of abſolute time can hardly amount to above 15 minutes, the change of place of the Earth's center will for that time be but ſmall; however, at the rate beforementioned, it will for 15 minutes affect the parallactic angle ſeen from Venus, by about $\frac{7}{100}$, of a ſecond; and the parallax of the Sun, by about $\frac{1}{400}$ part of the whole: but this proportional part will remain the ſame, whether the diſtance of meridians be ſuch as produce a greater or leſs difference of abſolute time than 15 minutes *.

* If an error of $\frac{1}{400}$ part of the whole may be ſuppoſed in the obſervation for determining the Sun's parallax by the tranſit

From what has been ſaid, I ſuppoſe it will appear, that the effects of the menſtrual parallax are worthy of conſideration; and that nothing has been yet executed, whereby it has received a determination ſufficiently accurate; for, in regard to obſervations upon the Sun, the whole quantity is too ſmall to be minutely obſerved in right aſcenſion: and with reſpect to the application to Mars and Venus, as ſuggeſted by Dr. Gregory, I do not know that any thing has been done; and indeed no wonder, as the theory of the motion of Mars and Venus has not been as yet ſo critically reduced to computation, as to render their parallaxes (though in themſelves much greater) deducible with equal certainty as that of the Sun.

What I therefore have now to propoſe, is a method of obſerving the menſtrual parallaxes of Mars and Venus, without laying any undue ſtreſs upon the theory of their motions.

The firſt opportunity of making an obſervation for this purpoſe, will be at the next oppoſition of Mars; which, according to the Nautical Almanack, will happen the 26th of October next, in the morning; I will therefore endeavour to illuſtrate this matter by taking that as an example.

The diſtance of Mars from the Earth will then be ſomewhat leſs than the mean diſtance, that is, as 1 to 2.2; and conſequently his double menſtrual parallax, according to Mr. Maſkelyne, will be near $31''$ in the point of oppoſition. Now, as the Moon

of Venus, a neglect of the menſtrual parallax may make it $\frac{1}{200}$ part of the whole.

will

will be at full, not above 12 hours preceding that oppoſition, the Moon will be nearly in the moſt favourable ſituation for the purpoſe.

For this end, let an accurate obſervation be made upon the place of Mars at the following times, viz. firſt, near the time of the new Moon, preceding Mars's oppoſition; or more properly at the neareſt opportunity, to the time of the Moon's oppoſition to Mars; which will happen in the night, between the 12th and 13th of October: ſecondly, let the place of Mars be obſerved when the Moon is neareſt her quartile with Mars; that is, between the 19th and 20th of the ſame month: thirdly, let an obſervation on Mars be made when the Moon is in conjunction with Mars, the neareſt to his oppoſition with the Sun; that is, between the 25th and 26th of ditto: fourthly, let Mars again be obſerved when the Moon has moved on to her quartile with Mars, viz. between the 31ſt of October, and 1ſt of November: and fifthly and laſtly, let the place of Mars be obſerved, when the Moon has again got to her oppoſition with Mars, which happens between the 7th and 8th of November.

Now it is manifeſt, that, when the Moon is in conjunction or oppoſition to Mars, the center of the Earth, the center of Mars, and the common center of gravity of the Earth and Mars, will be nearly in a right line, and conſequently, that an obſerver will then ſee Mars, in the ſame place in the heavens, as if the common center of gravity was the ſame as the center of the Earth; therefore, then the place of Mars will be unaffected by a menſtrual parallax; and ſuch will be the firſt, third, and fifth of the obſervations above propounded.

It is equally evident, that when the Moon is in quartile with Mars, and moving towards a conjunction, an obſerver, at the Earth's center, will ſee Mars more backward in the ecliptic, than if ſeen from the common center of gravity, by $15'' \frac{1}{2}$; and that, when the Moon is in her oppoſite quartile with Mars, and moving from her conjunction, that then an obſerver at the Earth's center, will ſee Mars advanced in his orbit more forward by $15'' \frac{1}{2}$, than if ſeen from the common center of gravity; and the one obſervation checqued with the other, will, according to a mean elliptic motion, differ by the quantity of $31''$; and ſuch will be the ſecond and fourth obſervations above propounded.

Now, from the firſt, third, and fifth, obſervations, three points of Mars's orbit will be given; which, by the help of the theory of Mars's motion in an elliptic orbit, whoſe aphelion, eccentricity, and nodes, are known ſufficiently near for this purpoſe; the intermediate places of Mars may be inferred with the requiſite degree of accuracy: and particularly, as the two intermediate obſervations, viz. the ſecond and fourth, will be nearly at equal intervals of time between the three others: from hence it follows, that the difference between the inferred, or computed places, at the quartiles, and the obſerved places at thoſe times, will be the menſtrual parallax required.

It is to be noted, that the times above ſpecified are the moſt favourable for the obſervation; and could thoſe be made uninterruptedly from weather, there would be the leſs occaſion for any other: but, as much as poſſible to prevent diſappointments of this kind, it will be right to begin the obſervations, a month preceding

ing making the proper obſervations, at the conjunctions, quartiles, and oppoſitions, of the Moon with Mars, which will be the means of ſupplying ſuch obſervations, as may happen to prove abortive before the oppoſition of Mars, and alſo, in caſe any of the obſervations to be made after that oppoſition ſhall prove deficient, the obſervations may be carried on for a month or competent time afterwards. As a further ſecurity againſt diſappointments, as well as cheque, it will alſo be adviſable to make the proper obſervations, the night preceeding and ſubſequent to thoſe in which the quartiles, conjunctions, &c. happen; for, as the quantities will not differ conſiderably from thoſe obtained on the days ſpecified, with proper allowances they may be brought in ſupport and confirmation of the former.

In like manner, when Venus is moving towards her inferior conjunction with the Sun, as will happen next year, the ſame obſervations may be made with reſpect to her; and continued for a neceſſary time, to get obſervations of the place of Venus; viz. the firſt, when the Moon is in conjunction or oppoſition with Venus: à ſecond, when the moon is in her quartile with Venus: a third, in conjunction or oppoſition: a fourth, when the moon is in her oppoſite quartile to the former: and a fifth, again in conjunction or oppoſition: the ſame opportunity will alſo offer when Venus is moving from her inferior conjunction with the Sun, and becomes a morning ſtar.

In regard to the obſervation of Venus, it is remarked by aſtronomers, that ſhe is to be ſeen with a good tranſit teleſcope, when ſhe is within a few degrees of the Sun; but, as ſhe is three times nearer the Earth, than

than the Sun's mean diſtance, when her elongation is 25° in the inferior part of her orbit, it is plain, that the neceſſary obſervations may eaſily be made, when her menſtrual parallax will be at a medium, three times greater than the Sun's; and conſequently amounting for the whole difference to 42″.

To avoid embarraſſment in deſcription, I have hitherto ſuppoſed, that all the obſervations are made in the meridian; in which caſe the right aſcenſions will be the ſame as they would appear from the center of the Earth; and conſequently, the planet's longitudes thence deduced, nearly the ſame: but 'tis eaſy to ſee, that if the quartile obſervations are made when the planets are conſiderably to the eaſt or weſt of the meridian, and ſo choſen, that the place of the obſerver be further diſtant from the common center of gravity, than the center of the Earth is from that center, that the baſe of the obſervations will be conſiderably enlarged. Thus, in our latitude, ſuppoſing that the quartile obſervations are made four hours before and four hours after the planet paſſes the meridian, this will produce an enlargement of the baſis by one of the Earth's ſemidiameters: and as the whole baſe or diameter of the epicycle comes out, according to Mr. Maſkelyne, no more than 1.6 of the Earth's ſemidiamers; the baſe will, according to this method, come out 2.6; and conſequently, at the next oppoſition, the menſtrual parallax of Mars will be thereby enlarged to 50″, the greateſt to 62″ $\frac{1}{2}$, and that of Venus at a mean, to 74″ $\frac{1}{2}$.

It muſt however be acknowledged, that no kind of obſervations of the places of the planets are of equal validity with thoſe taken with the beſt inſtruments in the

the meridian; thoſe taken with micrometers perhaps not excepted: for however accurately ſmall diſtances can be meaſured by the micrometer of Mr. Dollond, yet, as theſe meaſures can hardly be reduced to the ecliptic, without having the difference of declination or right aſcenſion from other means (except two ſtars making ſomewhat near a right angle with the planet ſhould appear within the field of view at once); and as in all theſe caſes the rectification of the places of the ſtars themſelves ultimately depends on meridian obſervations we may perhaps be allowed to ſay, that in the moſt favourable caſes of the micrometer, the determinations thence to be drawn, are not ſuperior to meridian obſervations, and in leſs favourable caſes, muſt be inferior: however, as the micrometer obſervations out of the meridian give an opportunity of repetition as often as we pleaſe; and the obſervations for rectification of the ſtars concerned, can be repeated in the meridian, as often as we pleaſe alſo; it muſt be equally allowed, that when theſe kind of obſervations are taken, not too near the horizon, when proper ſtars offer for this purpoſe, and the whole ſkilfully managed; theſe kind of obſervations fall but little ſhort of thoſe taken immediately in the meridian. I cannot therefore heſitate to recommend, that the quartile obſervations be taken out of the meridian, as well as in it: in the firſt place, by Dollond's micrometer, if ſtars offer in proper poſitions; and if not, ſecondly, by taking differences of right aſcenſion and declination between the planet and the ſtars, by the common micrometer, in caſe proper ſtars offer themſelves for this pupoſe: but as it frequently happens, that no proper ſtars offer themſelves to micrometers

of

of either kind; and this is ſtill more likely to happen in the obſervations of Venus, which will be chiefly in the day light; I beg leave to offer (what to me is) a new method of obſervation out of the meridian; and which, though I eſteem it not equal to micrometer obſervations of either kind, I apprehend will fall ſo little ſhort thereof, and prove ſo much ſuperior to any other method now in practice in theſe caſes, that I hope I ſhall on this occaſion be excuſed, in giving a particular deſcription thereof: but, as it is a general method of obſerving out of the meridian, I ſhall reſerve it by way of appendix.

In the next obſervation of Mars, it has been ſtated, that, in the meridian obſervations alone, the menſtrual parallax, according to the ſmalleſt eſtimation, may be expected to amount to $31''$ in longitude; which, turned into right aſcenſion, will make about $2''$ of time: now, if it may be allowed, that a well-practiſed obſerver can take the time of a tranſit to $\frac{1}{4}$ part of a ſecond, over a ſingle wire, if he has three wires, or more, as uſual, the mean of the three ſhould be within $\frac{1}{12}$ part of a ſecond; or within $\frac{1}{24}$ part of the whole quantity in queſtion: it is however a matter of chance, whether the mean of three may or may not be within $\frac{1}{3}$ part of the whole; and as equal errors may be committed in the obſervations of the tranſits of the ſtars, wherewith the right aſcenſions of the planets in queſtion are compared; which it is an equal chance, whether they tend to correct or increaſe the errors committed in the former; yet if, as has already been propoſed, the obſervations are continued for two or three months, inſtead of one; and obſervations, taken the day preceding and ſubſequent

to

to the days of conjunction, quartile, and oppoſition; and this as well out of the meridian as in it; we can hardly doubt but that, if the weather ſhould favour, ſo many cheques would be formed, that, from the next oppoſition of Mars alone, the affair may be brought within a 24th part of the whole; and, if to this be added the force of ſuch determinations, as may be drawn from obſervations on Venus, before and after her tranſit over the Sun next year, it can hardly be doubted, but that thoſe three will bring us within a ſingle ſecond of a degree, ſubtended from the neareſt planet; and theſe concluſions will be further ſtrengthened by future obſervations; as two years will ſcarcely paſs without affording one or more opportunities of this kind.

As I meant not to embarraſs myſelf with exact computations, I have conſtantly ſuppoſed the diſtance of the common center of gravity from the center of the Earth, to be a fixed quantity; whereas it will vary in the ſame proportion as the Moon's diſtance varies; but, as this and many other *minutiæ* will properly enter the computation, when the obſervations are made, I muſt beg leave to refer them to the learned in this ſcience.

Auſthorpe, April 17, 1768.

J. Smeaton.